UNSTEADY-STATE THERMAL SELF-FOCUSING OF PULSES

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In absorbing media, one of the principal mechanisms for the appearance of self-focusing can be heating of the medium by radiation, leading to a change in the dielectric constant of the substance ε. Since the greatest change in the temperature takes place in the region of a maximal dose of radiation, i.e., in the central region of the laser beam (or pulse), for $d\varepsilon/dT > 0$ there is the possibility of the appearance of self-focusing [1]. Thermal self-focusing was first observed in sapphire and glasses [2]. Subsequent experiments showed that thermal self-focusing is possible in a rather broad class of substances. Thus, the investigation of the development of damages in media with the passage of laser radiation [2, 3] leads to the conclusion that many of the experimental results can be explained only starting from the concept of thermal self-focusing. We note that the latter is observed for a relatively small power of the radiation; the threshold is usually 10-100 W. The relative contribution of different mechanisms of nonlinearity (thermal, strictive, Kerr) is discussed, for example, in [4]. The theory of steady-state thermal self-focusing has been rather well developed [5]. However, in a broad range of powers and durations of the pulse of radiation, the steady-state approach is insufficient and a study must be made of the unsteady-state problem. This is discussed in the present article. Here the following limitations of the general problem are postulated: 1) the absorption is assumed to be linear (as a rule, this approximation is sufficient, since the principal contribution to the absorption is that of single-photon processes [2]); 2) the absorption is assumed to be relatively small (such that, in the length of the self-focusing z_f, the absorbed energy will be much less than the initial energy of the pulse). The present article sets forth the results of both an analytical consideration and a numerical modeling.

1. Basic Equations

We represent the dielectric permittivity in the form of the sum

$$\varepsilon = \varepsilon_0 + i\varepsilon_1 + \varepsilon_T,$$

where ε_0 and ε_1 are the real and imaginary parts of the constant (i.e., not depending on the field of the radiation) component of ε; ε_T is a thermal correction, which depends on the change in the temperature and density of the medium. In a linear approximation,

$$\varepsilon_T = (\partial\varepsilon/\partial\rho)_T\delta\rho + (\partial\varepsilon/\partial T)_\rho\delta T, \tag{1.1}$$

where δT and $\delta\rho$ are, respectively, the deviations of the temperature and the density from their equilibrium values T_0 and ρ_0. The relative contribution of each of the terms of (1.1) to ε_T can be equal, depending on the parameters of the medium and the pulse. Correspondingly, the characteristic times of the processes, for which these terms are significant, differ. If the pulse is sufficiently short, then in a time τ (the length of the pulse) the density of the medium cannot change appreciably, in distinction from the molecular polarizability, described by the second term in (1.1). Then the term $(\partial\varepsilon/\partial\rho)_T\delta\rho$ can be neglected. For long pulses, a change in ε, connected with a change in the density, can appear; since this change usually far exceeds the change in the molecular polarizability, in (1.1) only the first term need be taken into consideration. Thus, it is expedient to consider only the limiting cases:

$$\varepsilon_T = (\partial\varepsilon/\partial\rho)_T\delta\rho \equiv \varepsilon_2\delta\rho; \tag{1.2}$$

$$\varepsilon_T = (\partial\varepsilon/\partial T)_\rho\delta T \equiv \varepsilon_3\delta T. \tag{1.3}$$

In the general case, the sign of $d\varepsilon/dT$ may be different depending on the temperature, the length of a wave of the laser radiation, and other factors. For self-focusing it is required that $d\varepsilon/dT > 0$, which is observed for many substances under ordinary conditions (different types of glasses, silicon, sapphire, and others [1, 2, 5]). In principle, there is the possibility of the appearance of self-focusing, both with a direct dependence of the dielectric permittivity on the temperature (1.3) and with temperature compression [6] (expansion) of the medium (1.2). A widely known example is as follows: Water at $T < 4°C$ contracts with heating,

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and $\partial\varepsilon/\partial\rho > 0$, which gives $d\varepsilon/dT > 0$. For a cold plasma $\partial\varepsilon/\partial\rho < 0$; therefore, heating followed by scattering of the plasma also leads to $d\varepsilon/dT > 0$.

We first consider the case where the mechanism of the change in the dielectric permittivity with the temperature is temperature compression (or expansion) of the substance (1.2).

The equation for a slowly varying complex amplitude E(r, z, t) of the field of the laser radiation is obtained from the Maxwell equations, as usual representing the intensity of the field $\mathscr{E}$ in the form

$$\mathscr{E} = (1/2)(Ee^{i\omega t+ikz} + \text{c.c.}).$$

Taking into consideration that E(r, z, t) changes only slightly in the distance of a wavelength $1/k$ and after a time on the order of a period $1/\omega$, we obtain

$$2ik[(1/v_{gr})E_t + E_z] + \Delta_\perp E + (k^2/\varepsilon_0)(i\varepsilon_1 + \varepsilon_T)E = 0, \tag{1.4}$$

where $v_{gr} = c/\sqrt{\varepsilon_0}$.

To (1.4) we add equations determining the change in the density $\delta\rho$, limiting ourselves everywhere to terms linear with respect to $\delta\rho$ and δT [4]:

$$\partial\delta\rho/\partial t + \rho_0\nabla_\perp v = 0,$$
$$\rho_0\partial v/\partial t = -\nabla_\perp\delta p,$$

where $p = p(\rho, S)$ is an equation of state connecting the pressure p, the density ρ, and the entropy S. Since

$$\delta p = \frac{\partial p}{\partial\rho}\delta\rho + \frac{\partial p}{\partial S}\delta S,$$

from this we obtain

$$\frac{\partial^2\delta\rho}{\partial t^2} - c_s^2\Delta_\perp\delta\rho = \left(\frac{\partial p}{\partial S}\right)_\rho\Delta_\perp S,$$

where c_s is the speed of sound in the medium; $\Delta_\perp = (1/r)(\partial/\partial r)[r(\partial/\partial r)]$.

We obtain an equation which is insufficient to close the system using the equation of the balance of the thermal energy Q (without taking account of the thermal conductivity)

$$\rho_0\frac{\partial Q}{\partial t} = \frac{c\sqrt{\varepsilon_0}}{8\pi}\frac{k\varepsilon_1}{\varepsilon_0}|E|^2$$

and the thermodynamic relationship $T = \partial Q/\partial S$. Summing up, we have

$$\rho_0 T_0\frac{\partial S}{\partial t} = \frac{c\sqrt{\varepsilon_0}}{8\pi}\frac{k\varepsilon_1}{\varepsilon_0}|E|^2.$$

If the thermal conductivity is taken into consideration, we can obtain

$$\frac{\partial S}{\partial t} - \chi\Delta_\perp S = \frac{c\sqrt{\varepsilon_0}}{8\pi\rho_0 T_0}\frac{k\varepsilon_1}{\varepsilon_0}|E|^2,$$

where χ is the coefficient of thermal conductivity.

Combining the equations written, we have the system

$$\begin{gathered}2ik\left(\frac{1}{v_{gr}}E_t + E_z\right) + \Delta_\perp E + \frac{k^2}{\varepsilon_0}(i\varepsilon_1 + \varepsilon_T)E = 0,\\ \frac{\partial S}{\partial t} - \chi\Delta_\perp S = \frac{c\sqrt{\varepsilon_0}}{8\pi\rho_0 T_0}\frac{k\varepsilon_1}{\varepsilon_0}|E|^2,\\ \frac{\partial^2\delta\rho}{\partial t^2} - c_s^2\Delta_\perp\delta\rho = \left(\frac{\partial p}{\partial S}\right)_\rho\Delta_\perp S.\end{gathered} \tag{1.5}$$

In the derivation of (1.5), the second derivatives with respect to the longitudinal coordinate were everywhere neglected. It is essential to take them into consideration only for pulses for which the longitudinal dimension is comparable to the characteristic transverse dimension a. Assuming that $a \sim 0.1$–0.01 cm, we obtain the result that neglect is not justified for pulses of length $\tau \sim 10^{-11}$–10^{-12} sec or shorter.

In the case where thermal self-focusing is due to the term $\delta T\,\partial\varepsilon/\partial T$, in analogous fashion we arrive at the system

$$2ik\left(\frac{1}{v_{gr}}E_t + E_z\right) + \Delta_\perp E + \frac{k^2}{\varepsilon_0}(i\varepsilon_1 + \varepsilon_3\delta T)E = 0,$$
$$\delta T_t - \chi\Delta_\perp\delta T = \frac{c\sqrt{\varepsilon_0}}{8\pi\rho_0 c_p}\frac{k\varepsilon_1}{\varepsilon_0}|E|^2. \quad (1.6)$$

We introduce the notation for the characteristic times: τ_S is the characteristic time of the hydrodynamic scattering of the substance; $\tau_S \sim a/c_S$ and is generally 10^{-7}-10^{-8} sec; τ_T is the time of the "departure" of heat from the focusing region; $\tau_T \sim 0.1$ sec; τ_f is the time of the development of self-focusing, determined by the parameters of the medium and the pulse. Depending on the relationships between these times, different simplifications of system (1.5) or (1.6) are possible. We shall consider some limiting cases.

2. Adiabatic Self-Focusing

Let the length of a pulse and the time of self-focusing be much greater than the time of the hydrodynamic scattering of the substance from the focal region. Then the process of self-focusing can be regarded as adiabatic. Substituting $\delta\rho_{tt} = 0$ from (1.5) into the wave equation, we arrive at the system

$$2ik\left(\frac{1}{v_{gr}}E_t + E_z\right) + \Delta_\perp E + \frac{k^2}{\varepsilon_0}\left(i\varepsilon_1 + \frac{\partial\varepsilon}{\partial\rho}\delta\rho\right)E = 0,$$
$$\delta\rho_t - \chi\Delta_\perp\delta\rho = \frac{c\sqrt{\varepsilon_0}}{8\pi\rho_0 T_0 c_s^2}\frac{\partial p}{\partial S}\frac{k\varepsilon_1}{\varepsilon_0}|E|^2, \quad (2.1)$$

which, with an accuracy up to the definitions, coincides with (1.6).

Thus, in the above sense, the case $\varepsilon_T = \delta T\partial\varepsilon/\partial T$ is equivalent to the limiting case $\varepsilon_T = \delta\rho\partial\varepsilon/\partial\rho$. If the quantities r, z, t, δT, and E are measured in the units a, l, $2k^2a^2/v_{gr}$, $\varepsilon_0/k^2a^2\varepsilon_3$, and $(8\pi\rho_0 c_p/k^2a^2\varepsilon_3)^{1/2}$, respectively, then, the system (1.6) assumes the form

$$i(u_t + \eta u_z) + \Delta_\perp u + (i\nu + \sigma T)u = 0,$$
$$T_t - \mu\Delta_\perp T = 2\nu|u|^2, \quad (2.2)$$

where all the variables are dimensionless. Here $\nu = k^2a^2\varepsilon_1/\varepsilon_0$, $\eta = 2ka^2/l$, $\mu = 2\chi k/\rho_0 c_p v_{gr}$, and c_p is the heat capacity. The dimensionless coefficient σ is introduced for convenience in the numerical experiments. This coefficient is introduced by the replacement $t \to \sigma t$, $z \to \sigma z$, $r \to \sqrt{\sigma}r$, $\nu \to \nu\sigma$ or the replacement $u \to \sqrt{\sigma}u$, $T \to \sigma T$.

Let us write the equivalent transformations leading to the dimensionless form (2.2) of system (2.1).

If we assume that $a \sim 0.1$ cm and $\chi \sim 0.1$-1 cm^2/sec, then the thermal conductivity is obviously significant only for pulses with a duration of $\tau > 0.01$ sec (this limit is made more precise below).

We first consider shorter pulses, i.e., we neglect the thermal conductivity and set $\mu = 0$. We go over to the coordinates ξ, z',

$$t = \xi + z/\eta, \quad z = z'$$

and introduce $u = W\exp(-\nu z/\eta)$.

We then arrive at the equations

$$i\eta W_z + \nabla_\perp W + \sigma TW = 0,$$
$$T_\xi = 2\nu|W|^2\exp(-2\nu z/\eta). \quad (2.3)$$

Let r_0 be a quantity characterizing the transverse dimension of the beam. We shall seek the solution of (2.3) for $r < r_0$, i.e., in the preaxial region, in the form

$$W = \psi e^{i\varphi}, \quad \psi = \psi_0(\xi)\exp\left[-r^2/r_0^2 f^2(\xi, z)\right]/f(\xi, z),$$
$$\varphi = \alpha(\xi, z) + \beta(\xi, z)r^2/2, \quad (2.4)$$

where $\psi_0(\xi)$ is the envelope of the input pulse. The statement (2.4) assumes that the pulse in a transverse cross section, if only near the axis, has a Gaussian form, which does not follow from the starting equations. The following general statement would be more natural:

$$\psi = \frac{\psi_0(\xi)}{f(\xi, z)}V\left(\frac{r}{r_0 f(\xi, z)}\right),$$

flowing out of the fact that the first of Eqs. (2.3) has the integral of the motion

$$I = \int |W|^2 r dr.$$

However, (2.4) reproduces quantitatively the general character of the solution, and we shall start from this representation. From (2.3) it follows that ψ and φ satisfy the equations

$$\eta\psi_z^2 + \frac{2}{r}\frac{\partial}{\partial r}\left(r\psi^2\frac{\partial\varphi}{\partial r}\right) = 0,$$

$$\eta\varphi_z + \frac{1}{\psi}\Delta_\perp\psi + \varphi_r^2 - \sigma T = 0,$$

$$T = 2\nu\int\limits_0^\xi \psi^2 \exp(-2\nu z/\eta)\, d\xi.$$

We substitute here the expressions (2.4) and make use of the fact that we are considering the preaxial region, i.e., we expand the expressions obtained in powers of $(r/r_0)^2$ and limit ourselves to the first terms of the expansion. Then, equating to zero the coefficients with first powers of $(r/r_0)^2$, we arrive at the equations for f and β:

$$\beta = \eta f_{zz}/2f, \quad \eta^2\frac{f_{zz}}{4f} - \frac{4}{r_0^4 f^4} + \frac{4\sigma\nu}{r_0^2}\int\limits_0^\xi \frac{\psi_0^2(y)\exp(-2\nu z/\eta)}{f^4(y, z)}\, dy = 0. \tag{2.5}$$

For the appearance of self-focusing, it is required that in the second of these equations the last term, assuring the constriction of the pulse to the axis, exceed the second term, connected with diffraction spreading. Assuming a sufficiently smooth form of ψ_0, we obtain from this the condition for the appearance of self-focusing:

$$\sigma\nu J\xi\exp(-2\nu z/\eta) > 1, \tag{2.6}$$

where $J = \bar{\psi}_0^2 r_0^2$; $\bar{\psi}_0^2$ is the mean value of ψ_0^2.

Thus, J has the sense of the power of the pulse. Since we are considering the case of weak absorption, we neglect the exponent in (2.6) and arrive at the orientation of self-focusing in the form

$$I > I_{cr} = 1/\sigma\nu, \tag{2.7}$$

where $I = J\tau$ is the energy of a pulse. Of course, due to the large number of approximations and extrapolations, the criterion (2.7) has a rough character. Numerical experiments have shown that we require an additional coefficient of the excess in (2.7) on the order of 3-5, depending on the form of the initial pulse.

As is well known, in the theory of steady-state self-focusing, the critical quantity, determining the possibility of the appearance of self-focusing, is the power of the beam. As can be seen, in the unsteady-state theory, an analogous role is played by the energy of a pulse.

Let us consider Eq. (2.5) in more detail. If f depends only weakly on z, then the term $\eta^2 f_{zz}/f$ can be neglected. Now, setting $f(\xi = 0) = 1$, we obtain

$$f(\xi) = \exp\left(-\frac{\sigma\nu r_0^2}{4}\int\limits_0^\xi \psi_0^2(y)\, dy\right) = \exp(-I(\xi)/4I_{cr}), \tag{2.8}$$

where $I(\xi) = r_0^2\int\limits_0^\xi \psi_0^2(y)\, dy$ has the sense of the energy passing through a cross section with the longitudinal coordinate ξ.

The second of Eqs. (2.5) has the form of the equation of motion of a nonlinear oscillator under the action of two "forces" of different sign, i.e., diffraction and a nonlinear force. For small values of ξ, i.e., at the limiting front of the pulse, where the nonlinear term is small, there will be ordinary differential spreading. The same region of ξ where the condition $I(\xi) > I_{cr}$ is satisfied is embraced by self-focusing conditions. In this region, there is the possibility of the appearance of oscillations of f around the equilibrium position, determined from (2.8).

Let us evaluate the dimension of the oscillations. Setting $f = f_0 + \delta f$, where f_0 is the equilibrium position, and assuming that $\delta f \ll 1$ and $\delta f \sim \exp(iz/\lambda)$, we obtain from (2.5)

$$\frac{\eta^2}{\lambda^2} \approx -\frac{16}{r_0^4}\left(-\frac{1}{5f^4} - \frac{4}{5}\right).$$

whence, for $f_0 \leq 1$,

$$\lambda \sim \frac{1}{3} f_0^2 \eta r_0^2.$$

From relationship (2.8) we have a simple evaluation for the minimal amplitude and the minimal transverse dimension of the pulse:

$$u_{\max} \sim \psi_0 \exp(I/4I_{cr}), \quad r_{\min} \sim r_0 \exp(-I/4I_{cr}). \tag{2.9}$$

Thus, the ratio I/I_{cr} mainly determines the dynamics of thermal self-focusing.

Oscillations of f and, consequently, of the amplitude and the radius of a pulse, arise for the condition

$$\lambda < \eta\tau, \quad \frac{1}{3} f_0^2 r_0^2 < \tau,$$

i.e., for sufficiently small values of f and I_{cr}/I. Up to this moment, the pulse will be focused as a whole. Setting $f_0 \sim \exp(-I/4I_{cr})$, we obtain an evaluation for the minimal dimension of the oscillations:

$$\lambda_{\min} \sim \eta r_0^2 \exp(-I/2I_{cr}).$$

As can be seen from (2.8), for $I \gg I_{cr}$, in the process of self-focusing the pulse is noticeably constricted to the axis, and the thermal conductivity must then be taken into consideration, even for short pulses. From (2.9) it follows that the thermal conductivity is significant with the propagation of pulses of a length

$$\tau \gtrsim r_{\min}^2/\mu = r_0^2 \exp(-I/2I_{cr})/\mu.$$

Under real conditions, the condition $I \gg I_{cr}$ is easily satisfied. We note that evaluations analogous to those given above are valid also for the propagation of powerful pulses in a medium with inertial Kerr nonlinearity, since the basic equation in this case has the form (2.2) for $\mu = 0$, and the role of ν is played by a quantity inverse to the relaxation time of the nonlinearity [7].

The system (1.6) was investigated numerically. The discussion was carried through in natural Euler variables in the following statement: A pulse of radiation of Gaussian profile

$$u(r, z = 0, t) = A \exp(-r^2/r_0^2 - t^2/\tau^2) \tag{2.10}$$

falls on the boundary of the medium z = 0 and, being absorbed in the medium with z > 0, is focused. The initial temperature is taken equal to zero, and the initial field has the form

$$u(r, z, t = 0) = 0{,}1 \exp(-r^2/r_0^2 - z^2/\tau^2\eta^2). \tag{2.11}$$

The initial moment of time t = 0 is so selected that the initial and boundary conditions (2.10) and (2.11) will be consistent.

The results of numerical modeling confirm sufficiently well the evaluations made above for the dynamics of the evolution of a pulse.

Self-focusing has a threshold with respect to the energy of a pulse. The value of the threshold is in agreement with the evaluation (2.7). If the ratio I/I_{cr} has a value of 1-3, then, after attaining a maximum of the amplitude, the pulse spreads out rapidly. If the condition $I > I_{cr}$ is satisfied with a margin of 3-5 times or more, this spreading takes place very slowly, so that it is possible to speak of stabilization of the pulse. The level of stabilization is close to that predicted by the relationship (2.9).

The dependence of the value of $u_{max} = \max|u|$ on the absorption coefficient ν for fixed parameters of the medium and the pulse A = 1, $r_0 = 3$, $\tau = 2$, $\sigma = 80$, and $\eta = 0.5$ is given in Fig. 1.

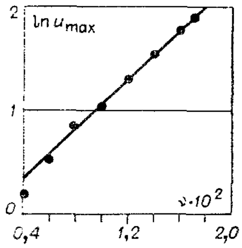

Fig. 1

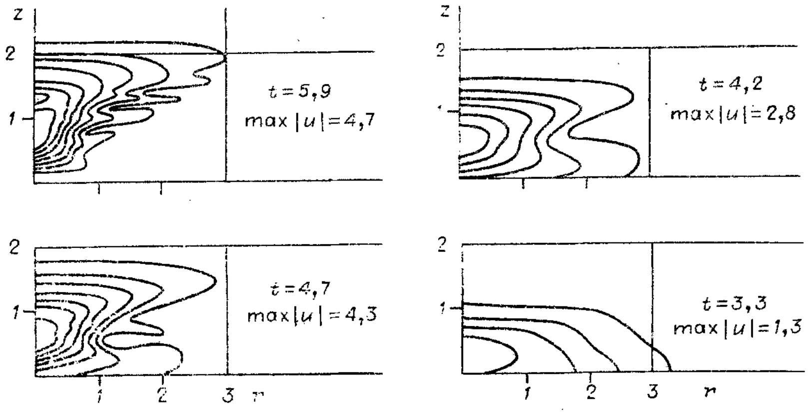

Fig. 2

After the maximal amplitude has attained a certain value, characteristic oscillations develop in the rear part of the pulse. Thus, for the parameters of Fig. 1, oscillations arise starting with an amplitude of ~3.

Figure 2 gives the level lines $|u| = 0.3, 0.5, 0.7, 1, 1.5, 2$, and 3 for a pulse at successive moments of time t with $\nu = 0.014$.

Since, in dimensional variables, the longitudinal dimension of the amplitude is much greater than the transverse, we can speak of a tendency toward the breaking-up of the pulse into individual bunches ("multifocus structure" [8]). The evolution of each such pulse takes place according to the following scheme: The focus is split by constriction into two parts; the leading part moves forward along the pulse, while the rear part moves in the opposite direction (focuses A and B, Fig. 3, $\nu = 0.014$). The focus moving ahead undergoes the same evolution as the initial focus. In the developed stage of the process, oscillations of approximately identical amplitude are formed on the profile of the pulse (see Fig. 3). At the leading front, new maxima are formed continuously, running backward along the profile of the pulse and vanishing in the tail part.

With an increase in the energy of the pulse I, the dimensions of oscillations decrease, their number increases. This circumstance constitutes an obstacle to the attainment of large values of u_{max}. With small values of $I \sim 1 - 3I_{cr}$, after attaining a maximal amplitude, the pulse spreads out without the formation of oscillations.

To demonstrate convincingly that it is precisely the energy of the pulse which mainly determines the dynamics of the process of self-focusing, the parameters r_0, σ, and ν were varied over a wide range, but in such a way that the value of the ratio I/I_{cr} would not change. It was found that, with an accuracy of ~5%, the maximally attainable amplitude of the pulse under these circumstances does not change. The length of the self-focusing rises considerably with a rise in the transverse dimension of the initial pulse.

Oscillations of the temperature were observed in the calculation, but they were not great.

The system (1.6) was considered numerically and for $\mu \neq 0$, i.e., taking account of thermal conductivity. The effect of the thermal conductivity rises with a rise in the energy of the pulse. Figure 4 gives axial profiles of the temperature for different values of the parameters σ and μ for t = 8 [1) $\sigma = 4$, $\mu = 0$; 2) $\sigma = -1$, $\mu = 0$; 3) $\sigma = 4$, $\mu = 0$; $\nu = 0.014$].

We note that oscillations of the field also arise with strictive and Kerr mechanisms of the self-focusing of the pulses [7].

3. Supersonic Self-Focusing

If the development time of self-focusing τ_f is less than the time of the hydrodynamic scattering of the substance τ_s (or is comparable with it), it is necessary to consider the total problem, i.e., the system (1.5). In the limiting case $\tau_f \ll \tau_s$ (supersonic self-focusing), in the last of these equations the term $c_s^2 \Delta_\perp \delta\rho$ can be dropped.

In dimensionless variables, system (1.5) assumes the form

$$i(u_t + \eta u_z) + \Delta_\perp u + (i\nu + \sigma\rho)u = 0,$$
$$\rho_{tt} - \mu\Delta_\perp\rho = \varkappa\Delta_\perp S, \quad S_t = 2\nu|u|^2, \tag{3.1}$$

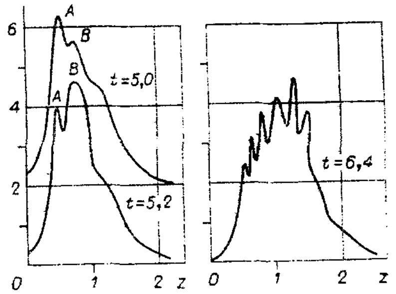

Fig. 3

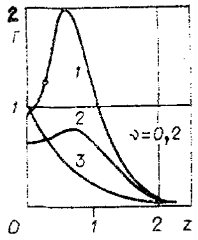

Fig. 4

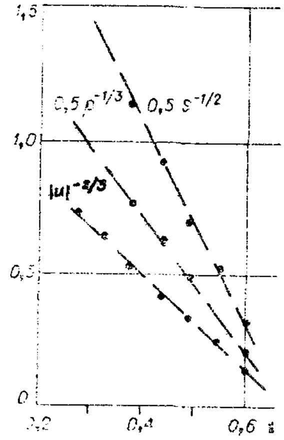

Fig. 5

where u is the amplitude of a pulse; ρ and S are, respectively, the perturbations of the density and the entropy; $\nu = k^2a^2\varepsilon_1/\varepsilon_0$; $\mu = 4c_S^2k^2a^2/v_{gr}^2$; $\eta = 2ka^2/l$. The values of r, z, t, u, ρ, and S are measured in the units a, l, $2ka^2/v_{gr}$, $(2\pi\rho_0T_0c^4/\varepsilon_0\varepsilon_2k^4a^4\partial p/\partial S)^{1/2}$, $\varepsilon_0/k^2a^2\varepsilon_2$, $c^2/(4k^4a^4\varepsilon_2\partial p/\partial S)$, respectively. The factors σ and $\varkappa$ are introduced, as before, for convenience in calculations by the replacement $u \to (\sigma\varkappa)^{1/2}u$, $\rho \to \sigma\rho$, $S \to \sigma S$.

Let us consider the case $\mu = 0$. For sufficiently long pulses (or bunches), it can be assumed that $\eta = 0$. Then (3.1) admits of an approximate self-similar statement:

$$u = \frac{1}{(t_0 - t)^{3/2}} V(r/(t_0 - t)^{3/2}),$$

$$\rho = \frac{1}{(t_0 - t)^3} R(r/(t_0 - t)^{3/2}), \quad S = \frac{1}{(t_0 - t)^2} Q(r/(t_0 - t)^{3/2}),$$

which can be demonstrated directly. It can be seen that, for long pulses, after a finite time t_0, a singularity (collapse) arises in the solution.

For the investigation of the self-focusing of pulses of finite length, we go over to "accompanying" coordinates $t = \xi - z/\eta$ and $z = z'$; we introduce $u = W\exp(-\nu z/\eta)$, after which we shall seek the solution in the form (2.4).

In the region $r \ll r_0$, for f we then obtain the equation

$$\frac{\eta^2 f_{zz}}{4f} - \frac{4}{f^4 r_0^4} + 24\sigma\nu\varkappa \int_0^\xi dy \int_0^y dx \int_0^x \frac{\psi_0^2(x')\,dx'}{r_0^4 f^6(x', z)} = 0. \tag{3.2}$$

From (3.2) it follows that self-focusing is possible if the last term in this equation exceeds the diffraction term $4/f^4r_0^4$. From this the condition for self-focusing has the form

$$\bar{f}^2 < \sigma\nu\varkappa\bar{\psi}_0^2\tau,$$

where $\bar{f}^2$ and $\bar{\psi}_0^2$ are the corresponding averaged values.

Since $f(\xi = 0) = 1$, then the criterion for self-focusing can be written in the form

$$I > I_{cr} = r_0^2/\tau^2\sigma\nu\varkappa, \tag{3.3}$$

where $I = \bar{\psi}_0^2 r_0^2\tau$ has the sense of the energy of a pulse.

The profile of the pulse enters explicitly into the evaluation (3.3), since r_0/τ is the ratio of the characteristic transverse and longitudinal dimensions.

Equation (3.2) is the equation of motion of a linear oscillator. Its "equilibrium point" f_0 is determined by the condition $f_{zz} = 0$, i.e.,

$$(1/f^4)_{\xi\xi\xi} = 6\sigma\nu\varkappa\psi_0^2(\xi)/f^6. \tag{3.4}$$

In the simplest case of a pulse of rectangular form

$$\psi_0(\xi) = \begin{cases} \psi_0 = \text{const}, & 0 < \xi < \tau, \\ 0, & \xi < 0,\ \xi > \tau \end{cases}$$

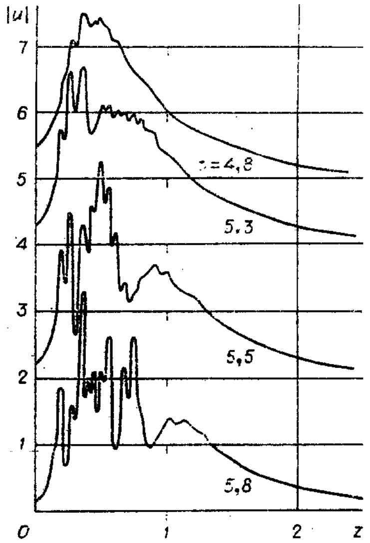

Fig. 6

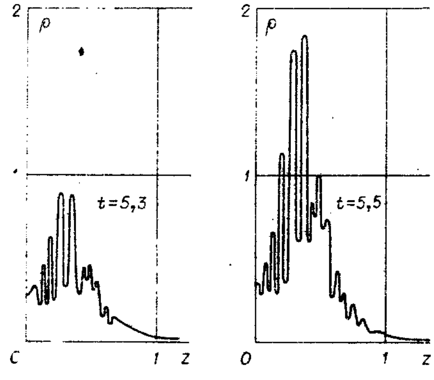

Fig. 7

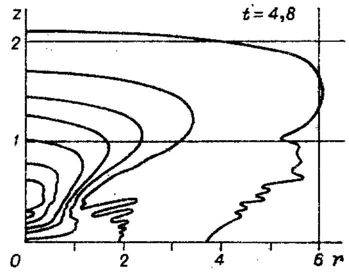

Fig. 8

we can find the explicit solution of Eq. (3.4):

$$f = \psi_0(\sigma\nu\varkappa/56)^{1/2}(\xi_0 - \xi)^{3/2}$$

(compare with the self-similar solution for long pulses). It follows from this that the change in the amplitude and transverse cross section of a pulse should have an explosive character as $\xi \to \xi_0$, i.e., with an approach to the tail part of the pulse. The quantity ξ_0 has the sense of the coordinate of collapse. The extremal values of the amplitude and radius of a pulse of length τ can be evaluated in the following manner:

$$|u|_{\max} \sim \psi_0'(\xi_0 - \tau)^{3/2}, \quad r_{\min} \sim r_0(\xi_0 - \tau)^{3/2}.$$

Let us evaluate the characteristic dimension of the oscillations of the value of f. Setting $f \sim f_0 + \delta f$, and assuming that $\delta f \ll 1$, $\delta f \sim \exp(iz/\lambda)$, from (3.2) for $\xi \sim \xi_0$ we obtain

$$\lambda \sim \frac{1}{3} f_0^2 r_0^2 \eta,$$

i.e., the minimal dimension of the oscillations

$$\lambda_{\min} \sim \frac{1}{150}(\xi_0/\tau - 1)^3 \eta r_0^2 I/I_{\mathrm{cr}}.$$

For pulses of nonrectangular form, the qualitative picture, i. e., the explosive character of the self-focusing, should be retained.

If $\mu \neq 0$, it is essential to take account of the hydrodynamic scattering. As follows from the self-similar solution, as $\xi \to \xi_0$ the term $\mu\Delta_\perp\rho$ rises as $(\xi_0 - \xi)^{-6}$, while, at the same time, $\rho_{tt} \sim (\xi_0 - \xi)^{-5}$.

We give the results of a numerical investigation of system (3.2). Pulses of Gaussian form were considered:

$$u(r, z = 0, t) = A\exp(-r^2/r_0^2 - t^2/\tau^2),$$

falling on the boundary z = 0 with the initial conditions

$$\rho(r, z, t = 0) = \rho_t(r, z, t = 0) = S(r, z, t = 0) = 0,$$
$$u(r, z, t = 0) = 0{,}1\exp(-r^2/r_0^2 - z^2/\tau^2\eta^2).$$

The time is reckoned from the moment when the initial and boundary conditions are in agreement.

The behavior of long pulses $\tau = \infty$ is close to the predicted self-similar law, as can be seen from Fig. 5. Self-similar conditions are naturally attained in a time much less than the absorption time of a pulse $1/\nu$, i.e., for $\sigma\nu\varkappa > 1$.

With the consideration of the total system for finite pulses, the parameter σ was varied. With small values of σ, the amplitude, after attaining a maximum, decreases and the pulse spreads out. With a rise in σ, oscillations appear in the tail part of the pulse. With sufficiently large values of σ, in the developed stage of self-focusing, the pulse is, so to speak, divided into two parts: a tail part, with a strongly oscillating amplitude of the intensity of the field, and a leading part, with weak oscillations. The dimension of the oscillations decreases sharply with an approach to the tail part of the pulse, as can be seen in Fig. 6, which shows the field at the axis of the pulse at different moments of time.

With a rise in σ, i.e., with a decrease in the development time of self-focusing, the geometric dimensions of the oscillations decrease sharply, which is a consequence of the explosive character of the process.

Figure 7 gives the distributions of the density at the axis of a pulse at two moments of time. The results shown in Figs. 6 and 7 were obtained with the parameters $A = 1$, $\tau = 2$, $r_0 = 3$, $\sigma = 50$, $\nu = 0.01$, $\varkappa = 2$, and $\eta = 0.5$.

A sharp rise in the maximally attainable amplitude starts from $\sigma = 50$. The strongly oscillating part of the pulse moves weakly ahead; there is a capture of the field by the well of the density, which has a strongly oscillating profile. The oscillations in the profile of S are weakly expressed. In Fig. 8, which gives the spatial distribution of the field with these same parameters at the moment $t = 4.8$, there can also be clearly seen the separation of the pulse into parts with strong and weak oscillations. The lines in Fig. 8 correspond to the levels $|u| = 0.1$, 0.3, 0.5, 0.7, 1, 1.5, and 2.

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